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Transient Thermal Analysis of Convective-Radiative Moving Fin under the Influences of Magnetic Field and Time-dependent Boundary Condition

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ABSTRACT

In this paper, unsteady thermal scrutiny of radiative-convective moving fin considering the influences of magnetic field and time-dependent boundary conditions is explored via Laplace transform method. The analytical solutions obtained are employed in the investigation of the impacts of Hartmann number, Peclet number, radiative and convective parameters on the transient thermal performance and effectiveness in the moving fin. The research outcomes establish that an increase in convective and porosity terms generates a corresponding increase in the fin's heat transfer rate. This consequently augments the fin's efficiency. Correspondingly, an increase in increases the magnitude of temperature distribution within the fin. It is also found that increasing the results in an increase in material mobility rate. Meanwhile, the exposure period of the material to its surrounding environmental conditions diminishes while fin losses more surface heat, hence the temperature of the fin intensifies. Finally, an increase in the fin's internal heat generation and thermal conductivity reduces heat transfer rate. Thus, the controlling terms of the fin during operation should be prudently selected to make sure that it retains its principal function of heat removal from the main surface.

1. Introduction

Enhancing heat transfer in thermal engines, electronic devices, mechanical practices and chemical procedures can be efficiently and passively accomplished through applications of fins and spines. The applications of fins to many industrial and engineering components have activated so many research works [2-18], just to mention a few. In the bid of theoretical investigation, the thermal damage problems and heat transfer enhancement by the extended surfaces, the controlling thermal models of the passive devices are always nonlinear. Although, there are various numerical and approximate-analytical schemes that have been used to solve the thermal problems [6-27]. In a recent work, Darvishi et al [28] studied steady state thermal performance in convective-radiative

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porous radial fin while in the same year, Hoshyar [29] adopted HPM to developed series solution to steady state thermal performance of longitudinal fin with inconstant internal heat generation. In the following year, Sobamowo [30] applied GMWR to some simple but highly accurate analytical solutions for thermal performance of fin under variable internal heat generation and thermal conductivity. Also, with the use of various approximate analytical methods and the effects of magnetic term on steady state thermal performance of solid and porous fins were explored by Hashar et al. [31], Oguntala et al. [32], Patel and Meher [33]. Additionally, Sobamowo [34] examined with FEM the optimum thermal design and performances of cooling fins under radiative and convective conditions. The study of thermal behavior of continuous moving surfaces such as extrusion, hot rolling, glass sheet or wire drawing, casting, powder metallurgy techniques for the fabrication of rod and sheet have become an area of increasing research interests. In the processes such as rolling of strip, hot rolling, glass fiber drawing, casting, extrusion, drawing of sheets and wires, there is usually the presence of heat exchange between surrounding and the stationary or moving material as depicted in Fig. 1.

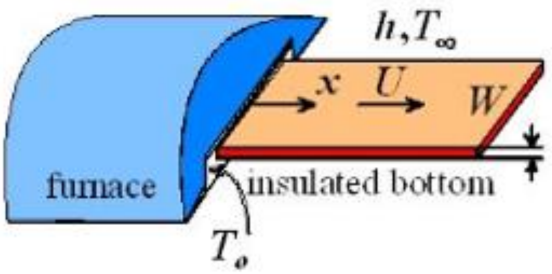


Fig. 1 Schematic diagram of rolling and extrusion

Since the schematic depicted in figure 1 satisfies the approximate working condition of a heat exchanging device, they can be modeled as fins moving uninterruptedly. Due to these adaptable and wide areas of applications, there have been extensive research works on the continuous moving fins. Moreover, in industrial processes, control of cooling rate of the sheets is very important to obtain desired material structure. As a result, numerous works on thermal investigation of moving fins have been offered in previous studies [35-42]. Aziz and Lopez [43] presented the numerical investigation of the convective-radiative moving fin. Torabi et al. [44] utilized DTM for analyzing continuously moving fin losing heat through both convection and radiation and having inconstant thermal conductivity. Various heat transfer techniques in variable thermal conductivity moving fin with and without heat generation have been presented [45-48]. Sun and Ma [49] used collocation spectral method to theoretically investigate the same problem. Singh et al. [50] used wavelet collocation approach for studying and understand convective-radiative traveling fins with varying thermal conductivities. With the application of simplex search method, Ranjan [51] scrutinized thermal performance of convective-conductive varying thermal conductivity fins. Yinusa and Sobamowo applied integral transform method to obtain the dynamic and stability responses of a nanotube in thermal and pressurized environments [52]. Recently, Oguntala et al. considered the thermal analysis of functionally graded longitudinal fin using integral transform. The obtain solution in their research was applied to improved electronic packaging. The review of the past studies shows that the analytical study of nonlinear transient heat transfer analysis in extended surfaces have not extensively been presented in literature. Moreover, the obvious advantages of generating exact analytical solutions to the nonlinear problems are very much important. However, to the best of the authors' knowledge, exact heat transfer analysis of a moving convective-radiative porous fin under temperature reliant thermal conductivity and subjected to magnetic effect hasn't been carried out. Such solutions provide proper physical insights and effective predictions to extended surfaces' thermal performances. Therefore, in this study, using Laplace transformation, exact analytical solution is developed for such problem with time-dependent boundary conditions. The developed symbolic thermal models are employed for investigating the influences of radiative term, convective term, Hartmann and Peclet numbers on the transient thermal performance, effectiveness in the moving fin, and other thermal geometric and thermo-physical fin properties.

2. Problem formulation

Consider a longitudinal straight fin with inconstant thermal conductivity and in a convective-radiative environment at temperature T_∞ and convective co-efficient *h* as in Fig.1. The fin is exposed to uniform magnetic field that is applied in y-direction. Assuming that the extended surface is porous, isotropic, homogeneous and saturated, i.e having a single-phase fluid with constant thermo-physical properties. There is a thermal equilibrium between the fluid and the solid. It is taken that heat transfer along fin length is one-dimensional. The prime surface is perfectly in thermal contact with the fin base and no heat gain or loss through the tip of the fin.

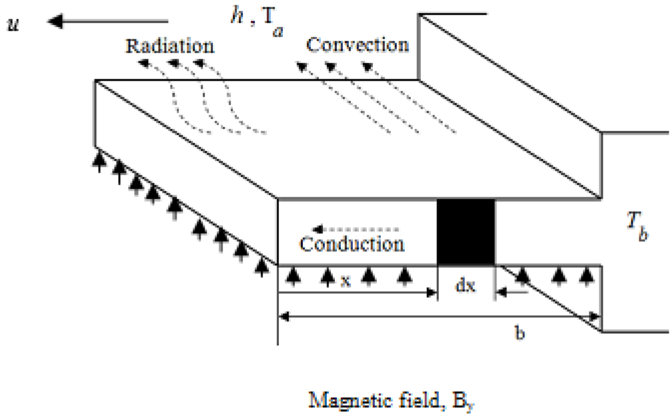


Fig. 2 Schematic of the convective-radiative moving longitudinal straight fin.

Using the energy balance, thermal model of the fin is developed as

$$\frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right]+\frac{4\sigma}{3\beta_R}\frac{\partial}{\partial x}\left(\frac{\partial T^4}{\partial x}\right)-\frac{h(T-T_a)}{\delta}-\frac{\sigma\varepsilon(T^4-T_a^4)}{\delta}-A_s\frac{\sigma_m(T)B_o^2u^2}{A_{cr}}=\rho c_p\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x} \tag{1}$$

Material thermal conductivity can be stated as being in linear relationship with temperature;

$$k\left(T\right)=k_a[1+\gamma(T-T_a)] \tag{2}$$

Substituting Eq. (2) into Eq. (1), we have

$$\frac{\partial}{\partial x}\left[[1+\gamma(T-T_a)]\frac{\partial T}{\partial x}\right]+\frac{4\sigma}{3\beta_R k_a}\frac{\partial}{\partial x}\left(\frac{\partial T^4}{\partial x}\right)-\frac{h(T-T_a)}{k_a\delta}-\frac{\sigma\varepsilon(T^4-T_a^4)}{k_a\delta}-A_s\frac{\sigma_m\left(T\right)B_o^2u^2}{k_aA_{cr}}=\frac{\rho c_p}{k_a}\frac{\partial T}{\partial t}+\frac{u}{k_a}\frac{\partial T}{\partial x} \tag{3}$$

The initial condition is given as

$$t=0,\quad T=T_\infty,\qquad 0\le x\le b \tag{4}$$

with boundary conditions

$$t>0,\quad x=0,\quad T=T_b(1-\beta^{'}t),\qquad \beta^{'}<0 \tag{5a}$$

$$t>0,\quad x=b,\quad \frac{\partial T}{\partial x}=0 \tag{5b}$$

When there is a small deferential in temperature within the material during the heat flow, the term T^4 may be stated as linear temperature function. Therefore, we have

$$T^4=T_a^4+4T_a^3\left(T-T_a\right)+6T_a^2\left(T-T_a\right)^2+...\cong 4T_a^3T-3T_a^4 \tag{6}$$

On substituting Eq. (6) into Eq. (3), we arrived at;

$$\frac{\partial^2 T}{\partial x^2}+\frac{16\sigma}{3\beta_R k_a}\frac{\partial^2 T}{\partial x^2}-\frac{h(T-T_a)}{k_a\delta}-\frac{4\sigma\varepsilon T_a^3(T-T_a)}{k_a\delta}-A_s\frac{\sigma_m\left(T\right)B_o^2u^2}{k_aA_{cr}}=\rho c_p\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x} \tag{7}$$

On introducing the following dimensionless parameters in Eq. (8) into Eq. (7),

$$\theta = \frac{T - T_a}{T_b - T_a}\ Ra = \frac{gk\beta\left(T_b - T_a\right)b}{\alpha \nu k_r}\ Rd = \frac{4\sigma_{st}T_a^3}{3\beta_R k_a}\ Nr = \frac{4\sigma_{st}b^2T_a^3}{k_a}\ Ha = \frac{\sigma_{mo}B_0^2u^2b^2}{k_a A_{cr}},\ X = \frac{x}{b} \tag{8}$$

$$Bi_e = \frac{h_e b}{k_a},\quad Bi_c = \frac{h_c b}{k_a},\quad M^2 = \frac{hb^2}{k_a\delta},\ Bi_{e,eff} = \frac{\left(h_e + \sigma\varepsilon\right)b}{k_a},\quad Bi_{ceff} = \frac{\left(h_c + \sigma\varepsilon\right)b}{k_a}$$

And taking magnetic term to linearly-dependent on temperature such as $\sigma_m\left(T\right) = \sigma_{mo}\left(T - T_a\right)$, gives the dimensionless form of the governing Eq. (7) as

$$\left(1 + 4Rd\right)\frac{\partial^2\theta}{\partial X^2} - M^2\theta - Nr\theta - Ha\theta = \zeta\frac{\partial\theta}{\partial\tau} + Pe\frac{\partial\theta}{\partial X} \tag{9}$$

The dimensionless forms of the boundary and initial conditions are

$$\tau = 0,\quad \theta = 0,\qquad 0 \le X \le 1 \tag{10}$$

$$\tau > 0,\quad X = 0,\quad \theta = 1 - \beta\tau,\qquad \beta < 0 \tag{11a}$$

$$\tau > 0,\quad X = 1,\quad \frac{\partial\theta}{\partial X} = 0 \tag{11b}$$

3. Method of Solution: Laplace Transform Method (LTM)

Laplace Transform Method is applied over the time in the transient thermal model. The merit of this method is that it can generate close form solutions to partial differential equations. Also, unlike numerical schemes, the method has no error due to discretization. Furthermore, the method generates close form solution with the controlling parameters adequately preserved. This makes the obtained solution using this method ready for parametric studies. Further merits associated with the employed method can be found in our previously published work [52]. The LTM of a real function *f(t)* and its inversion formulas are defined as

$$F(\mathrm{s}) = \int_0^\infty e^{-st} f(t)dt \tag{12a}$$

$$f(t) = \frac{1}{2\pi i}\int_{s-i\infty}^{s+i\infty} e^{-st} F(s)dt \tag{12b}$$

where $s=a+ib$ $(a, b \in R)$ is a complex number.

The integral transform scheme may be applied if we eliminate the first derivative in the governing equation and also, change the non-homogeneous equation to a homogenous equation. In order to achieve this, we adopt change of variables techniques.

Applying the transformation of the form

$$\theta\left(X,\tau\right) = \phi\left(X,\tau\right)e^{\left\{\frac{PeX}{2(1+4Rd)} - \left[\frac{Pe^2}{4\zeta(1+4Rd)} + \left(M^2+Nr+Ha\right)\right]\tau\right\}} \tag{13}$$

Eq. (9) reduces to

$$\frac{\partial\phi}{\partial\tau} = \frac{\partial^2\phi}{dX^2} \tag{14}$$

The boundary and initial conditions becomes

$$\tau = 0, \quad \phi = 0, \qquad 0 \le X \le 1 \tag{15}$$

$$\tau > 0, \quad X = 0, \quad \phi = (1-\beta\tau) e^{\left\{\left[\frac{Pe^2}{4\zeta(1+4Rd)} + \left(M^2+Nr+Ha\right)\right]\tau\right\}} \tag{16a}$$

$$\tau > 0, \quad X = 1, \quad \frac{\partial \phi}{\partial X} = -\frac{Pe\phi(1,\tau)}{2(1+4Rd)} \tag{16b}$$

With the application of Laplace transform to Eq. (14) and the boundary conditions, we have

$$\frac{\partial^2 \tilde{\phi}}{dX^2} - s\tilde{\phi} = 0 \tag{17}$$

The corresponding boundary conditions in Laplace domain are

$$s > 0, \quad X = 0, \quad \phi = \frac{1}{s - \left[\frac{Pe^2}{4\zeta(1+4Rd)} + \left(M^2+Nr+Ha\right)\right]} - \frac{\beta}{\left\{s - \left[\frac{Pe^2}{4\zeta(1+4Rd)} + \left(M^2+Nr+Ha\right)\right]\right\}^2} \tag{18a}$$

$$s > 0, \quad X = 1, \quad \frac{\partial \tilde{\phi}}{\partial X} = -\frac{Pe\tilde{\phi}(1,s)}{2(1+4Rd)} \tag{18b}$$

On solving Eq. (17) with the boundary conditions in Eq. (18), we have

$$\tilde{\phi}(X,s) = \left\{ \begin{array}{l} \dfrac{1}{s - \left[\frac{Pe^2}{4\zeta(1+4Rd)} + \left(M^2+Nr+Ha\right)\right]} \\ -\dfrac{\beta}{\left\{s - \left[\frac{Pe^2}{4\zeta(1+4Rd)} + \left(M^2+Nr+Ha\right)\right]\right\}^2} \end{array} \right\} \left\{ \frac{\left[\begin{array}{l} \sqrt{\frac{s}{(1+4Rd)}} cosh(1-X)\sqrt{\frac{s}{(1+4Rd)}} \\ +\frac{Pe}{2} sinh(1-X)\sqrt{\frac{s}{(1+4Rd)}} \end{array} \right]}{\left[\begin{array}{l} \sqrt{\frac{s}{(1+4Rd)}} cosh\sqrt{\frac{s}{(1+4Rd)}} \\ +\frac{Pe}{2} sinh\sqrt{\frac{s}{(1+4Rd)}} \end{array} \right]} \right\}$$

(19)

Eq. (19) could be written in exponential form as

$$\tilde{\phi}(X,s)=\left\{\begin{array}{l}\dfrac{1}{s-\left[\dfrac{Pe^2}{4\zeta(1+4Rd)}+\left(M^2+Nr+Ha\right)\right]}\\ -\dfrac{\beta}{\left\{s-\left[\dfrac{Pe^2}{4\zeta(1+4Rd)}+\left(M^2+Nr+Ha\right)\right]\right\}^2}\end{array}\right\}\left\{\begin{array}{l}e^{\left[-X\sqrt{\frac{s}{(1+4Rd)}}\right]}+e^{\left[-(2-X)\sqrt{\frac{s}{(1+4Rd)}}\right]}\\ -\left[\dfrac{\dfrac{Pe}{(1+4Rd)}}{\dfrac{Pe}{2(1+4Rd)}+\sqrt{\dfrac{s}{(1+4Rd)}}}\right]e^{\left[-(2-X)\sqrt{\frac{s}{(1+4Rd)}}\right]}\end{array}\right\}$$

(20)

On applying the inverse LTM of Eq. (20) and substituting the resulting solution in Eq. (13), the following solution are developed

$$\theta(X,\tau)=\left\{\begin{array}{l}\left\{\begin{array}{l}\dfrac{1}{2}\left[1+\beta\left(\dfrac{X\zeta}{Pe}-\tau\right)\right]erfc\left[\dfrac{X}{2}\sqrt{\dfrac{\zeta}{(1+4Rd)\tau}}-\sqrt{\left(\dfrac{Pe^2}{4\zeta(1+4Rd)}+\left(M^2+Nr+Ha\right)\right)\tau}\right]\\ +\dfrac{1}{2}\left[1-\beta\left(\dfrac{X\zeta}{Pe}+\tau\right)\right]exp\left(\dfrac{PeX}{(1+4Rd)}\right)erfc\left[\dfrac{X}{2}\sqrt{\dfrac{\zeta}{(1+4Rd)\tau}}+\sqrt{\left(\dfrac{Pe^2}{4\zeta(1+4Rd)}+\left(M^2+Nr+Ha\right)\right)\tau}\right]\end{array}\right\}\\ +\left\{\begin{array}{l}\dfrac{1}{2}\left[1-\beta\left(\dfrac{1-\sqrt{\zeta}}{1+\sqrt{\zeta}}\right)\left[\tau-\left(\dfrac{2-X}{Pe}\right)\zeta-\dfrac{4\beta(1+4Rd)\zeta^{\frac{3}{2}}}{\left[Pe\left(1+\sqrt{\zeta}\right)\right]^2}\right]\right]exp\left(-\dfrac{(1-X)Pe}{(1+4Rd)}\right)\\ \times erfc\left[\left(\dfrac{2-X}{2}\right)\left[\sqrt{\dfrac{\zeta}{(1+4Rd)\tau}}-\sqrt{\left(\dfrac{Pe^2}{4\zeta(1+4Rd)}+\left(M^2+Nr+Ha\right)\right)\tau}\right]\right]\end{array}\right\}\\ +\left\{\begin{array}{l}\dfrac{1}{2}\left[1-\beta\left(\dfrac{1+\sqrt{\zeta}}{1-\sqrt{\zeta}}\right)\left[\tau+\left(\dfrac{2-X}{Pe}\right)\zeta+\dfrac{4\beta(1+4Rd)\zeta^{\frac{3}{2}}}{\left[Pe\left(1-\sqrt{\zeta}\right)\right]^2}\right]\right]exp\left(\dfrac{Pe}{(1+4Rd)}\right)\\ \times erfc\left[\left(\dfrac{2-X}{2}\right)\left[\sqrt{\dfrac{\zeta}{(1+4Rd)\tau}}+\sqrt{\left(\dfrac{Pe^2}{4\zeta(1+4Rd)}+\left(M^2+Nr+Ha\right)\right)\tau}\right]\right]\end{array}\right\}\\ +\left(\dfrac{2\zeta}{\zeta-1}\right)\left\{\begin{array}{l}1-\beta\tau+\dfrac{4\zeta\beta(1+4Rd)}{Pe^2(\xi-1)}exp\left[\begin{array}{l}\dfrac{PeX}{2(1+4Rd)}-\dfrac{Pe^2\tau}{4\zeta(1+4Rd)}\\ +\dfrac{(2-X)\zeta Pe}{2(1+4Rd)}+\dfrac{Pe^2\tau}{4(1+4Rd)}\end{array}\right]\\ \times erfc\left[\left(\dfrac{2-X}{2}\right)\sqrt{\dfrac{\zeta}{(1+4Rd)\tau}}+\sqrt{\left(\dfrac{Pe^2}{4\zeta(1+4Rd)}+\left(M^2+Nr+Ha\right)\right)\tau}\right]\\ -\dfrac{16}{\sqrt{\pi}}\dfrac{\beta\zeta\sqrt{Pe\tau}}{\zeta-1}exp\left[\dfrac{PeX}{2(1+4Rd)}-\dfrac{Pe^2\tau}{4\zeta(1+4Rd)}-\dfrac{(2-X)^2\zeta}{4(1+4Rd)}\right]\end{array}\right\}\end{array}\right\}exp\left[-\left(M^2+Nr+Ha\right)\tau\right]$$

(21)

where

$$\zeta \neq 1$$

4. Results and Discussion

The developed analytical models are simulated in MATLAB and the results are given in Fig. 3-14. Also, parametric studies are carried out as presented and discussed.

Effects of *Pe* on temperature history in the moving fin are shown in Fig. 3 and 4. An increase in *Pe* resulted in increasing values of thermal distribution within the extended surface. This is expected because increasing *Pe* augments material motion and reduces exposure time to environment. Hence, fin temperature history intensifies.

Fig. 5-12 present the impacts of radiation, convective, Hartmann and radiative-conduction numbers on temperature histories within the moving extended surface.

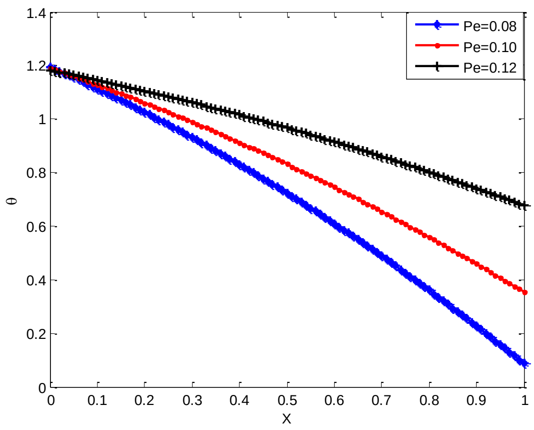


Fig. 3 Effects of Peclet numeber on the temeperature distribution along fin lenght

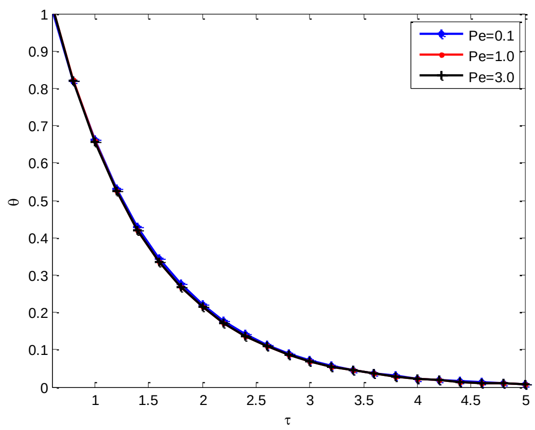


Fig. 4 Effects of Peclet numeber on the temeperature history

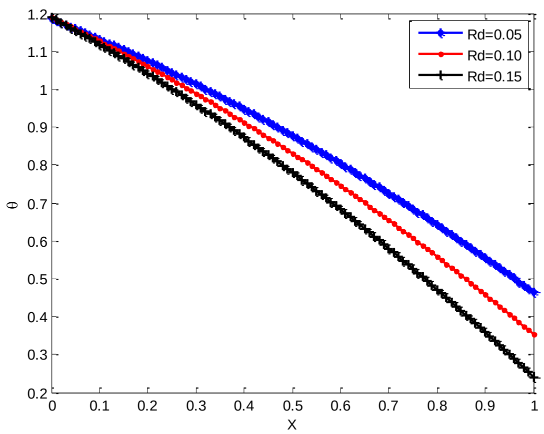


Fig. 5 Effects of Radiation numeber on the temeperature distribution

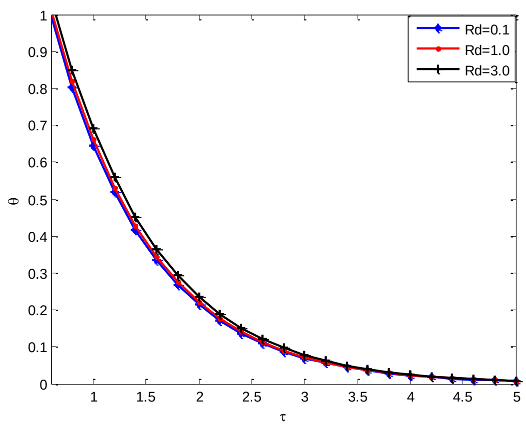
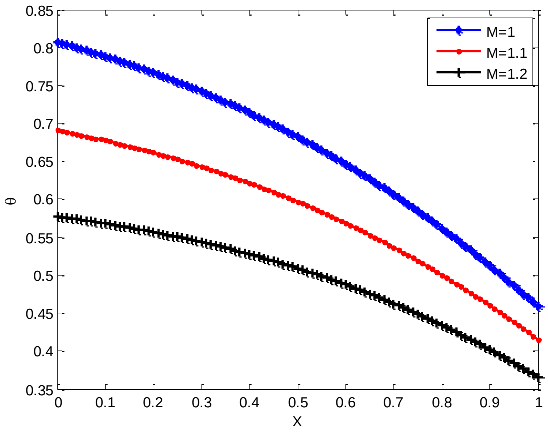


Fig. 6 Effects of Radiation numeber on the temeperature history

along the fin



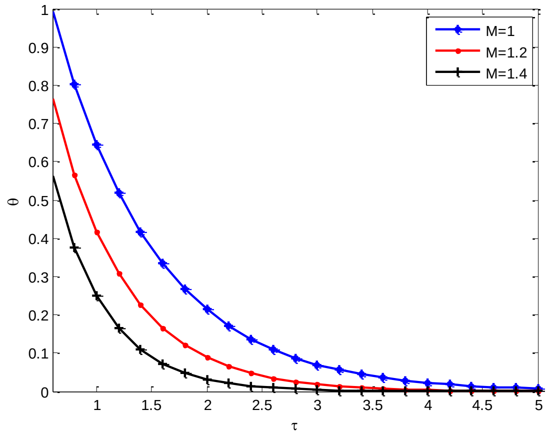
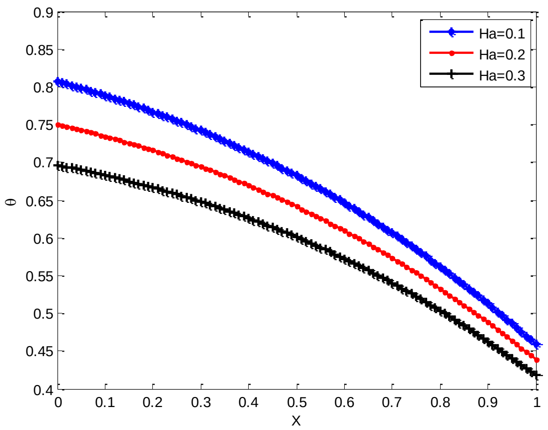


Fig. 7 Effects of convective numeber on the temeperature distribution along the fin

Fig. 8 Effects of convective numeber on the temeperature history



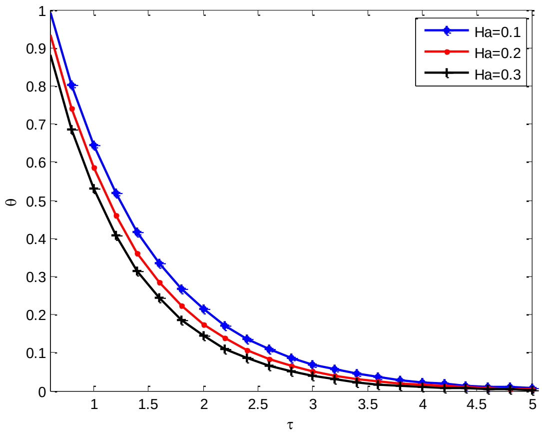
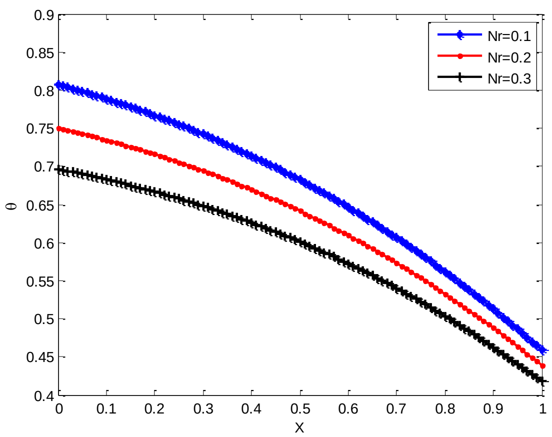


Fig. 9 Effects of Hartmann numeber on the temeperature distribution along the fin

Fig. 10 Effects of Hartmann numeber on the temeperature history



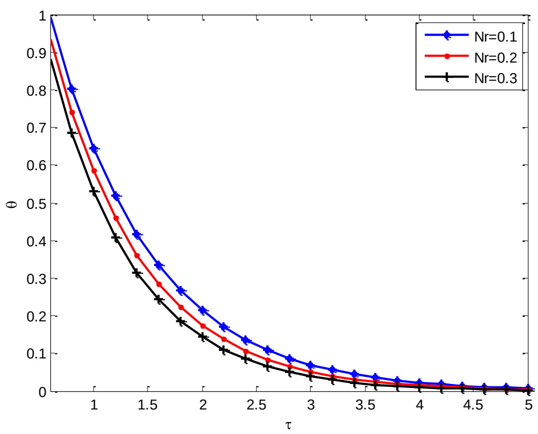


Fig. 11 Effects of Radiation parameter on the temeperature distribution along the fin

Fig. 12 Effects of radiation parameter on the temeperature history

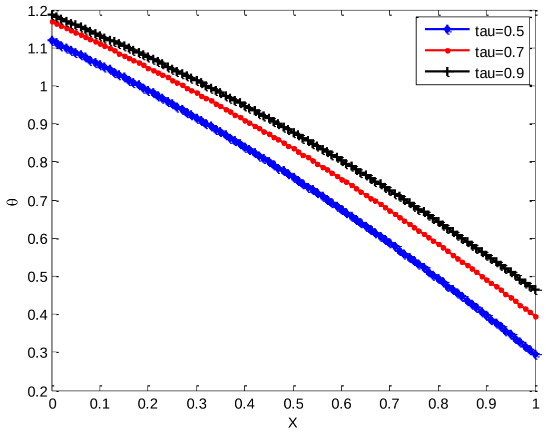


Fig. 13 Effects of dimensionless time on the temeperature distribution along the fin

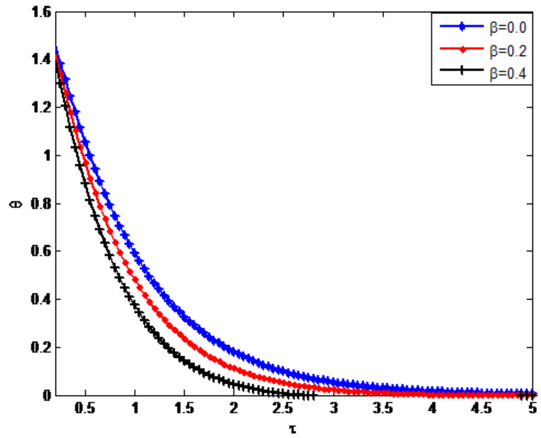


Fig. 14 Impacts of boundary condition parameter on the temeperature history

From the figures, it is demonstrated that as the radiation, convective, Hartmann and radiative-conduction parameters increase, fin temperature history declines rapidly. The swift reduction in temperature as a result of the increase in radiation, convective, Hartmann and radiative-conduction parameters is because as these parameters increase, more heat is lost from the fin because the heat transfer rate is enhanced and more cooling of the fin occurs which shows a decrease in the temperature profile. Consequently, fin thermal performance is increased.

Influence of dimensionless time on moving fin thermal distribution is shown in Fig. 13. The temperature history increases with increasing time value. This is expected because with increasing heat transfer rate, the solid fin conducts more heat, thus temperature increases. The influence of the time-dependent boundary condition parameter, β on the temperature history in the fin is presented in Fig. 14. It is clear from the figure that as the time-dependent boundary condition parameter is increased, the temperature distribution along the fin decreases. Physically speaking, the effect of an increase in the time-dependent boundary condition parameter, shows the decreases in the base temperature with time which results in decrease in the local temperature of the fin.

5. Conclusion

The present study considered heat transfer in porous moving fin with time dependent boundary condition under magnetic and non-constant thermal conductivity influences. Using LTM, analytical solution for such problem was obtained. Thereafter, parametric studies were carried out. The results revealed that increasing the radiative and convective terms improve fin efficiency and heat transfer rate. Also, an increase in *Pe* resulted in increasing values of thermal distribution within the extended surface. It was established that an augmentation in *Pe* augments material motion and reduces exposure time to environment. Hence, fin temperature history intensifies. Consequently, the controlling terms of the fin during operation should be prudently selected to make sure that it retains its principal function of heat removal from the main surface.

Nomenclature

A Fin cross sectional area, (m^2).
h Heat transfer coefficient, ($Wm^{-2}k^{-1}$).
T_b Temperature at the base of the fin, (K).
U Velocity of fin (m/s).
v_w velocity of fluid passing through the fin at any point (m/s).
w Width of the fin (m).
x Axial length measured from fin tip (m).
X Dimensionless axial length of the fin.

Greek Symbols
β Thermal conductivity parameter or non-linear parameter.
θ Dimensionless temperature.
ρ Density of the fluid(kg/m^3).

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