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Exact solutions of (2+1)-dimentional Sakovich equation using two well known methods

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ABSTRACT

Obtaining exact solutions of nonlinear differential equations is an applicable topic in physics and mathematics. The principal aim of the current research is to elicit exact solutions to the (2+1)-dimensional Sakovich equation employing two well-known methods including Kudryashov and G′/G expansion methods. Furthermore, several exact solutions from soliton solutions to periodic solutions to the equation are formally derived. Hence, the results are plotted to demonstrate the dynamics of the obtained solutions, and they indicate the existence of different wave structures in the governing model.

1. Introduction

For decades, a plethora of efforts has been made to exactly solve nonlinear partial differential equations (NLPDEs). Moreover, many methods with various strengths and weaknesses have been introduced to achieve the goal. Some of the most commonly used of them are first integral methods [1-4], ansatz method [5-9], G′/G expansion method [10], modified exp-function method [10,11], Kudryashov method [12-14], the functional variable method [15-17], and many others [18-25]. The present research focuses on two methods called Kudryashov and G′/G expansion methods to achieve exact solutions of NLPDEs. In both methods, the solution is expressed as a polynomial such that the solution process can be easily handled by symbolic computations [26-29].

The goal of the article is to achieve exact solutions of (2 + 1)-dimensional Sakovich equation. In the last two years, the following the (2 + 1)-dimensional second-order Sakovich equation has been introduced and studied by some researchers [30].

$$q_{xt} + q_{yy} + 2qq_{xy} + 6q^2 q_{xx} + 2(q_{xx})^2 = 0. \quad \textbf{(1)}$$

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The above-mentioned equation pertains to a category of second-order equations, quadratic in q_{xx}, that satisfy the Painlevé test for integrability and holds Korteweg-de-Vries (KdV)- sort multi-soliton solutions [1]. What sets this equation apart from the KDV equation is that this equation is not a third-order q_{xxx} scatter expression. Wazwaz and Sakovich have come up with good ideas for this newly developed equation and its remarkable properties [30,31]. In [32], the multi wave and interaction solutions of this equation are obtained by Lie symmetry analysis method.

The issues raised in other sections of the work are as succeeding. In part 2, the preliminary ideas of the Kudryashov method and the expansion of G '/ G will be discussed. In part 3, Kudryashov's method will be applied to achieve the exact solutions of the Sakovich equation. In Section 4, the G'/ G expansion method is used to discover exact solutions to the equation. Finally, a conclusion will be presented in Section 5.

2. Introducing the methods

In the current section, the general structure of Kudryashov and G′/G expansion methods to obtain the exact solutions of NLPDEs is briefly described.

2.1. kudryashov method

For describing the primary ideas of the Kudryashov method, the following NLPDE should be noted

$$p(q,\, q_t\, ,q_x,\, q_y, q_{xx},\, q_{tt},\, q_{yy},\, q_{xy}, \dots) = 0. \tag{2}$$

***Step 1*:** By changing the variable $\xi = \mu x + \delta y - wt,$

where δ , μ and w are nonzero constants, ***Eq. (2)*** stated as follows

$$Q(q, q', q'', ...) = 0 \tag{3}$$

Step 2: It is assumed that the solution of ***Eq. (2)*** has a solution as follows

$$q(\varepsilon) = \sum_{i=0}^{N} a_i R^i(\xi). \tag{4}$$

In which a_n, $n = 0,1,2,...,N$ $(a_N \neq 0)$ are unfamiliar parameters, and N can be achieved by balancing between the highest order derivatives and highest order nonlinear terms in ***Eq. (3)***, and $R(\xi)$ has the form

$$R(\xi) = \frac{4ae^{-\xi}}{4a^2 + \eta e^{-2\xi}}, \tag{5}$$

which fulfil the following equation

$$(R'(\xi))^2 = R^2(\xi)(1 - \eta R^2(\xi)). \tag{6}$$

we have

$$R''(\xi) = R(\xi) - \eta R^3(\xi),$$
$$R'''(\xi) = R'(\xi)(1 - 6\eta R^2(\xi)),$$
$$\vdots \qquad \textbf{(7)}$$

Step 3: Setting ***Eq. (4)*** into ***Eq. (3)*** results

$$F(R(\xi)) = 0, \qquad \textbf{(8)}$$

where $F(R(\xi))$ is a polynomial in $R(\xi)$. By considering the coefficient of each power of $R(\xi)$ in ***Eq. (8)*** equal to zero, a algebraic will be dreived for finding a_i, w, λ, *and* μ.

Step 4: By substituting the parameters obtained from step 3 in relation 4, the solution of the ***Eq. (2)*** will be resulted.

2.2. G′/G expansion method

To implement the G′/G expansion method, the following stages must be considered.

Step 1. By changing the variable

$$\xi = x + y - wt, \qquad \textbf{(9)}$$

where ω is constant, ***Eq. (2)*** will be stated as:

$$Q(q, q', q'', \ldots) = 0, \qquad \textbf{(10)}$$

which the superscripts denote the derivatives with respect to ξ.

Step 2. Suppose that the solution of ODE (***Eq. (10)***) can be written in G′/G as follows:

$$q(\xi) = \sum_{i=1}^{m} a_i \left(\frac{G'}{G}\right)^i, \qquad \textbf{(11)}$$

that $G = G(\xi)$ fulfill the second order linear ordinary differential equation in the following form

$$G'' + \lambda G' + \mu G = 0. \qquad \textbf{(12)}$$

Where $\alpha_i, \lambda,$ and μ are unknown parameters to be find with $\alpha_m \neq 0$. From ***Eq. (11)*** and ***Eq. (12)*** we derive

$$q' = -m\alpha_m\left(\frac{G'}{G}\right)^{m+1} - m\lambda\alpha_m\left(\frac{G'}{G}\right)^{m} + \cdots. \tag{13}$$
$$q'' = -m(m+1)\alpha_m\left(\frac{G'}{G}\right)^{m}\left(-\left(\frac{G'}{G}\right)^{2} - \lambda\left(\frac{G'}{G}\right) - \mu\right) + \cdots = m(m+1)\alpha_m\left(\frac{G'}{G}\right)^{m+2} + \cdots.$$
$$\vdots$$
$$q^{(n)} = (-1)^n m(m+1)...(\mathrm{m+n-1})\alpha_m\left(\frac{G'}{G}\right)^{m+n} + \cdots.$$

Step 3. For obtaining m, the homogeneous balance must be considered between the highest order derivatives and highest order nonlinear term in ***Eq. (10)***. Replacing ***Eq. (11)*** into ***Eq. (10)*** with considering ***Eq. (12)***, results in an algebraic equation including $\left(\frac{G'}{G}\right)^i$. Getting the coefficient of power of $\left(\frac{G'}{G}\right)^i$ to zero cause a system for obtaining unknown parameters.

Step 4. Placing over determine value in Step 3 in ***Eq. (11)***, and the general solutions of ***Eq. (12)***, the exact solutions of the ***Eq. (2)*** will be achieved.

3. Application Kudryashov method to the Sakovich equation

By applying the transformation $\xi = \mu x + \delta y - wt$, ***Eq. (1)*** convert to,

$$(\delta^2 - \mu w)\mathrm{q}'' + 2\mu\delta q q'' + 6\mu^2 q^2 q'' + 2\delta^4 (q'')^2 = 0. \tag{14}$$

By balancing principal, we derive $N = 2$. Therefore, the solution series is considered as follows.

$$q(\varepsilon) = a_0 + a_1 R(\xi) + a_2 R^2(\xi). \tag{15}$$

Where $R(\xi)$ satisfy in ***Eq. (6)***. Substituting ***Eq. (15)*** into ***Eq. (14)***, and considering the coefficient of $R(\xi)$ equal to zero, results in

$$\begin{aligned}
&\left(\delta^2 - \mu w\right) a_1 + 2\mu\delta a_0 a_1 + 6\mu^2 a_0^2 a_1 = 0 \\
&4\left(\delta^2 - \mu w\right) a_2 + 8\mu\delta a_0 a_2 + 2\mu\delta a_1^2 + 24\mu^2 a_0^2 a_2 + 12\mu^2 a_0 a_1^2 + 2\mu^4 a_1^2 = 0 \\
&-2\left(\delta^2 - \mu w\right)\eta a_1 - 4\mu\delta a_0 \eta a_1 + 10\mu\delta a_1 a_2 - 12\mu^2 a_0^2 \eta a_1 + 48\mu^2 a_0 a_1 a_2 \\
&\qquad + 6\mu^2\left(2 a_0 a_2 + a_1^2\right) a_1 + 16\mu^4 a_1 a_2 = 0
\end{aligned} \tag{16}$$

$$-6\left(\delta^2-\mu w\right)\eta a_2-12\mu\delta a_0\eta a_2-4\mu\delta a_1^2\eta+8\mu\delta a_2^2-36\mu^2 a_0^2\eta a_2-24\mu^2 a_0 a_1^2\eta + 24\mu^2\left(2a_0a_2+a_1^2\right)a_2+12\mu^2 a_1^2 a_2+2\mu^4\left(-4\eta a_1^2+16a_2^2\right)=0$$

$$-16\mu\delta a_1\eta a_2-72\mu^2 a_0 a_1\eta a_2-12\mu^2\left(2a_0a_2+a_1^2\right)\eta a_1+54\mu^2 a_1 a_2^2-56\mu^4 a_1\eta a_2=0$$

$$-12\mu\delta a_2^2\eta-36\mu^2\left(2a_0a_2+a_1^2\right)\eta a_2-24\mu^2 a_1^2 a_2\eta+24\mu^2 a_2^3+2\mu^4\left(4\eta^2 a_1^2-48\eta a_2^2\right)=0$$

$$48\eta^2\mu^4 a_1 a_2-84\eta\mu^2 a_1 a_2^2=0$$

$$72\eta^2\mu^4 a_2^2-36\eta\mu^2 a_2^3=0$$

Solving above system, leads to

$$a_0=-\frac{1}{6}\frac{4\mu^3+\delta}{\mu},\ a_1=0,\ a_2=2\mu^2\eta,\ w=\frac{1}{6}\frac{(16\mu^6+5\delta^2)}{\mu}. \quad \textbf{(17)}$$

So, the solutions of the sakovich equation can be acheined as follows

$$q(\varepsilon)=-\frac{1}{6}\frac{4\mu^3+\delta}{\mu}+32a^2\mu^2\eta\frac{\left(e^{-\delta y-\mu x+\frac{1}{6}\frac{(16\mu^6+5\delta^2)}{\mu}t}\right)^2}{\left(4a^2+\eta e^{-2\delta y-2\mu x+\frac{1}{3}\frac{(16\mu^6+5\delta^2)}{\mu}t}\right)^2}, \quad \textbf{(18)}$$

where a,η,δ and μ is a constant. The plots of solution (***Eq. (18)***) for various parameters are drawn in ***Figures 1, 2***. Considering $a=\frac{1}{2}$ and $\eta=1$, the solution (***Eq. (18)***) will be as follows

$$\begin{aligned} q(\varepsilon)&=-\frac{1}{6}\frac{4\mu^3+\delta}{\mu}+2\mu^2\sec h^2(-\delta y-\mu x+\frac{1}{6}\frac{(16\mu^6+5\delta^2)}{\mu}\mathrm{t}) \\ &=-\frac{1}{6}\frac{-8\mu^3+\delta}{\mu}-2\mu^2\tan h^2(-\delta y-\mu x+\frac{1}{6}\frac{(16\mu^6+5\delta^2)}{\mu}\mathrm{t}). \end{aligned} \quad \textbf{(19)}$$

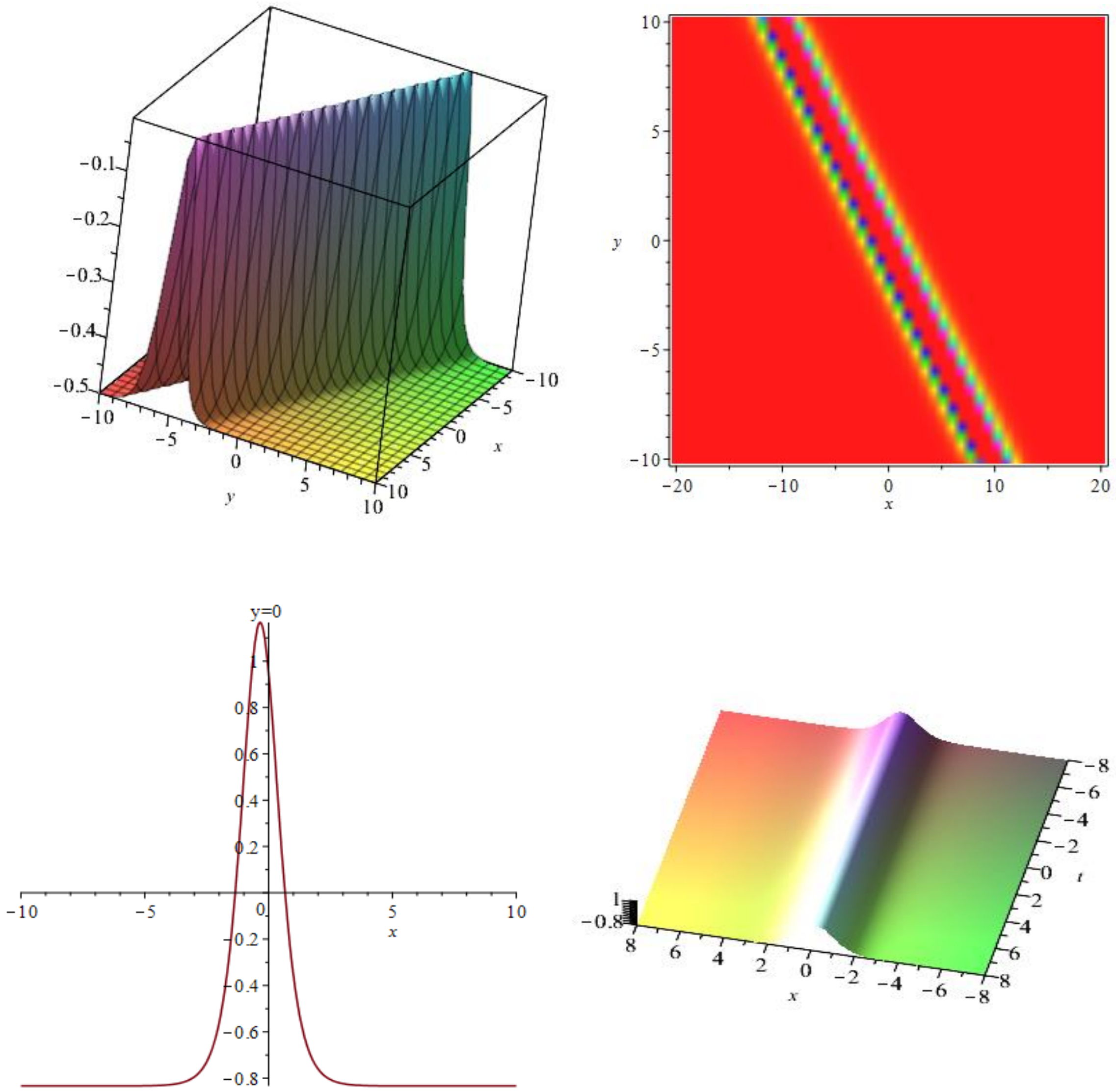

Figure 1. The plot of solution ***Eq. (18)*** for $\delta = 1, \mu = 1, a = 1, \eta = 1$ and $t = 0.1$

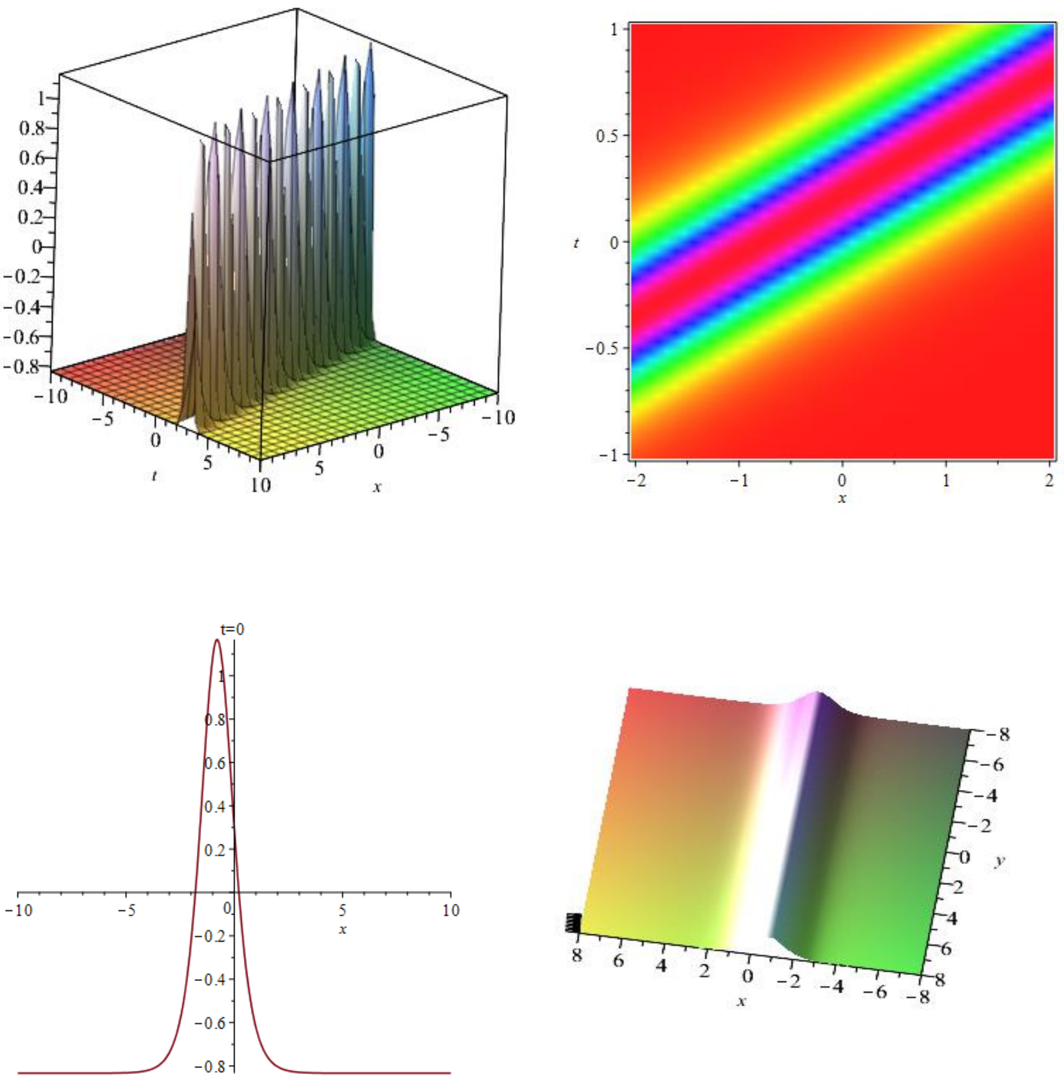

Figure 2. The plot of ***Eq. (18)*** for $\delta = 1, \mu = 1$ and $y = 0.1$

4. Application G′/G-expansion method to the Sakovich equation

By considering, $\xi = x + y - wt$ ***Eq. (1)*** convert to

$$(1-w)q'' + 2qq'' + 6q^2q'' + 2(q'')^2 = 0. \quad \textbf{(20)}$$

By balancing principal, we derive $m = 2$. Therefore, the solution series is considered as follows.

$$q(\xi) = \alpha_2\left(\frac{G'}{G}\right)^2 + \alpha_1\left(\frac{G'}{G}\right) + \alpha_0, \quad \alpha_2 \neq 0. \quad \textbf{(21)}$$

Substituting ***Eq. (21)*** into ***Eq. (20)***, and placing the coefficient of $\frac{G'}{G}$ equal to zero, results in system of equations.

$$\begin{aligned}
&(1-w)\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)+2\alpha_0\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)+12\alpha_0^2\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)\\
&\quad+2\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)^2=0
\end{aligned} \tag{22}$$

$$\begin{aligned}
&(1-w)\left(\lambda^2\alpha_1+6\lambda\mu\alpha_2+2\mu\alpha_1\right)+2\alpha_0\left(\lambda^2\alpha_1+6\lambda\mu\alpha_2+2\mu\alpha_1\right)+2\alpha_1\left(\lambda\mu\alpha_1\right.\\
&\quad\left.+2\mu^2\alpha_2\right)+12\alpha_0^2\left(\lambda^2\alpha_1+6\lambda\mu\alpha_2+2\mu\alpha_1\right)+24\alpha_0\alpha_1\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)\\
&\quad+4\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)\left(\lambda^2\alpha_1+6\lambda\mu\alpha_2+2\mu\alpha_1\right)=0
\end{aligned}$$

$$\begin{aligned}
&(1-w)\left(10\lambda\alpha_2+2\alpha_1\right)+2\alpha_0\left(10\lambda\alpha_2+2\alpha_1\right)+2\alpha_1\left(4\lambda^2\alpha_2+3\lambda\alpha_1+8\mu\alpha_2\right)\\
&\quad+2\alpha_2\left(\lambda^2\alpha_1+6\lambda\mu\alpha_2+2\mu\alpha_1\right)+12\alpha_0^2\left(10\lambda\alpha_2+2\alpha_1\right)+24\alpha_0\alpha_1\left(4\lambda^2\alpha_2\right.\\
&\quad\left.+3\lambda\alpha_1+8\mu\alpha_2\right)+12\left(2\alpha_0\alpha_2+\alpha_1^2\right)\left(\lambda^2\alpha_1+6\lambda\mu\alpha_2+2\mu\alpha_1\right)\\
&\quad+24\alpha_1\alpha_2\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)+4\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)\left(10\lambda\alpha_2+2\alpha_1\right)+4\left(\lambda^2\alpha_1\right.\\
&\quad\left.+6\lambda\mu\alpha_2+2\mu\alpha_1\right)\left(4\lambda^2\alpha_2+3\lambda\alpha_1+8\mu\alpha_2\right)=0
\end{aligned}$$

$$\begin{aligned}
&(1-w)\left(4\lambda^2\alpha_2+3\lambda\alpha_1+8\mu\alpha_2\right)+2\alpha_0\left(4\lambda^2\alpha_2+3\lambda\alpha_1+8\mu\alpha_2\right)+2\alpha_1\left(\lambda^2\alpha_1\right.\\
&\quad\left.+6\lambda\mu\alpha_2+2\mu\alpha_1\right)+2\alpha_2\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)+12\alpha_0^2\left(4\lambda^2\alpha_2+3\lambda\alpha_1+8\mu\alpha_2\right)\\
&\quad+24\alpha_0\alpha_1\left(\lambda^2\alpha_1+6\lambda\mu\alpha_2+2\mu\alpha_1\right)+12\left(2\alpha_0\alpha_2+\alpha_1^2\right)\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)\\
&\quad+4\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)\left(4\lambda^2\alpha_2+3\lambda\alpha_1+8\mu\alpha_2\right)+2\left(\lambda^2\alpha_1+6\lambda\mu\alpha_2+2\mu\alpha_1\right)^2\\
&\quad=0
\end{aligned}$$

$$\begin{aligned}
&6(1-w)\alpha_2+12\alpha_0\alpha_2+2\alpha_1\left(10\lambda\alpha_2+2\alpha_1\right)+2\alpha_2\left(4\lambda^2\alpha_2+3\lambda\alpha_1+8\mu\alpha_2\right)+72\\
&\quad\alpha_0^2\alpha_2+24\alpha_0\alpha_1\left(10\lambda\alpha_2+2\alpha_1\right)+12\left(2\alpha_0\alpha_2+\alpha_1^2\right)\left(4\lambda^2\alpha_2+3\lambda\alpha_1+8\mu\alpha_2\right)\\
&\quad+24\alpha_1\alpha_2\left(\lambda^2\alpha_1+6\lambda\mu\alpha_2+2\mu\alpha_1\right)+12\alpha_2^2\left(\lambda\mu\alpha_1+2\mu^2\alpha_2\right)+24\alpha_2\left(\lambda\mu\alpha_1\right.\\
&\quad\left.+2\mu^2\alpha_2\right)+4\left(\lambda^2\alpha_1+6\lambda\mu\alpha_2+2\mu\alpha_1\right)\left(10\lambda\alpha_2+2\alpha_1\right)+2\left(4\lambda^2\alpha_2+3\lambda\alpha_1\right.\\
&\quad\left.+8\mu\alpha_2\right)^2=0
\end{aligned}$$

$$\begin{aligned}
&12\alpha_2^2+72\left(2\alpha_0\alpha_2+\alpha_1^2\right)\alpha_2+24\alpha_1\alpha_2\left(10\lambda\alpha_2+2\alpha_1\right)+12\alpha_2^2\left(4\lambda^2\alpha_2+3\lambda\alpha_1\right.\\
&\quad\left.+8\mu\alpha_2\right)+24\alpha_2\left(4\lambda^2\alpha_2+3\lambda\alpha_1+8\mu\alpha_2\right)+2\left(10\lambda\alpha_2+2\alpha_1\right)^2=0
\end{aligned}$$

$$144\alpha_1\alpha_2^2+12\alpha_2^2\left(10\lambda\alpha_2+2\alpha_1\right)+24\alpha_2\left(10\lambda\alpha_2+2\alpha_1\right)=0$$

Solving above system, leads to

$$\alpha_0=-\frac{1}{6}\lambda^2-\frac{4}{3}\mu-\frac{1}{6},\alpha_1=-2\lambda,\ \alpha_2=-2,w=\frac{1}{6}\lambda^4-\frac{4}{3}\lambda^2\mu+\frac{8}{3}\mu^2+\frac{5}{6}. \quad \textbf{(23)}$$

By replacing ***Eq. (20)*** into ***Eq. (19)***, we have

$$q(\xi)=-2\left(\frac{G'}{G}\right)^2-2\lambda\left(\frac{G'}{G}\right)-\frac{1}{6}\lambda^2-\frac{4}{3}\mu-\frac{1}{6}, \quad \textbf{(24)}$$

where

$$\xi=x+y-(\frac{1}{6}\lambda^4-\frac{4}{3}\lambda^2\mu+\frac{8}{3}\mu^2+\frac{5}{6})t. \quad \textbf{(25)}$$

Putting the general solutions of ***Eq. (12)*** into ***Eq. (21)***, leads to the following general solution of ***Eq. (2)***:

When $\lambda^2-4\mu>0$,

$$q(\xi)=-\frac{\lambda^2-4\mu}{2}\left(\frac{A\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\right)+B\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\right)}{A\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\right)+B\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\right)}\right)^2+\frac{\lambda^2}{3}-\frac{4}{3}\mu-\frac{1}{6}. \quad \textbf{(26)}$$

If B=0, so

$$q(\xi)=-\frac{\lambda^2-4\mu}{2}\tanh^2\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\right)+\frac{\lambda^2}{3}-\frac{4}{3}\mu-\frac{1}{6}. \quad \textbf{(27)}$$

And when A=0, we derive

$$q(\xi)=-\frac{\lambda^2-4\mu}{2}\coth^2\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\right)+\frac{\lambda^2}{3}-\frac{4}{3}\mu-\frac{1}{6}. \quad \textbf{(28)}$$

When $\lambda^2-4\mu<0$,

$$q(\xi)=-\frac{4\mu-\lambda^2}{2}\left(\frac{A\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi+B\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi}{A\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi+B\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi}\right)^2+\frac{\lambda^2}{3}-\frac{4}{3}\mu-\frac{1}{6}. \quad \textbf{(29)}$$

If B=0, so

$$q(\xi)=-\frac{4\mu-\lambda^2}{2}\tan^2(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi)+\frac{\lambda^2}{3}-\frac{4}{3}\mu-\frac{1}{6}. \quad \textbf{(30)}$$

And when A=0, we derive

$$q(\xi)=-\frac{4\mu-\lambda^2}{2}\cot^2(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi)+\frac{\lambda^2}{3}-\frac{4}{3}\mu-\frac{1}{6}. \quad \textbf{(31)}$$

When $\lambda^2-4\mu=0$,

$$q(\xi)=-2\left(\frac{B}{A+B\xi}\right)^2+\frac{\lambda^2}{3}-\frac{4}{3}\mu-\frac{1}{6}. \quad \textbf{(32)}$$

The plot of the above solutions for different values of the parameters is drawn in ***Figures 3-5***.

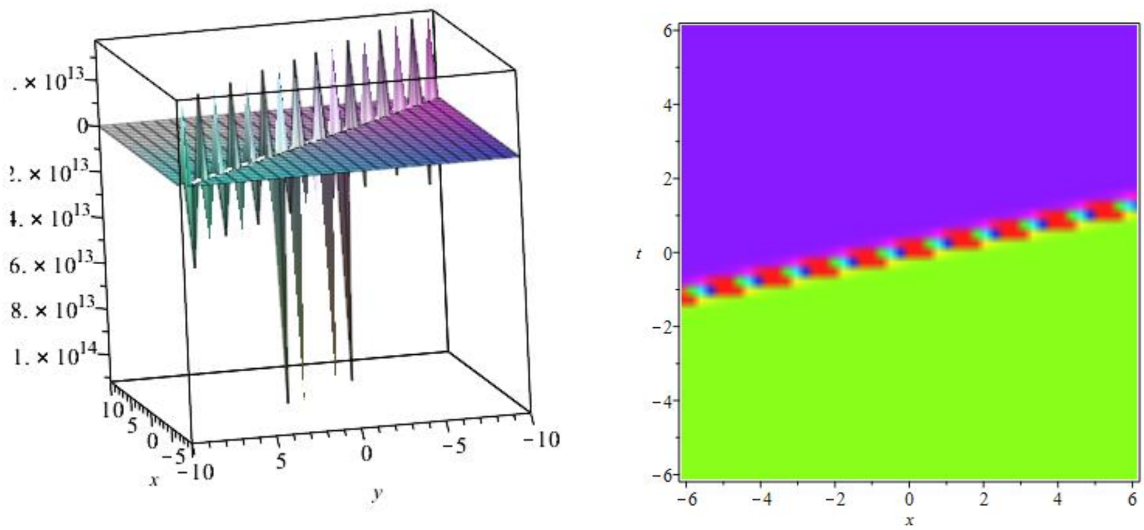

Figure 3. plots of ***Eq. (26)*** with $A=1, B=2, \lambda=3, \mu=1, y=0$.

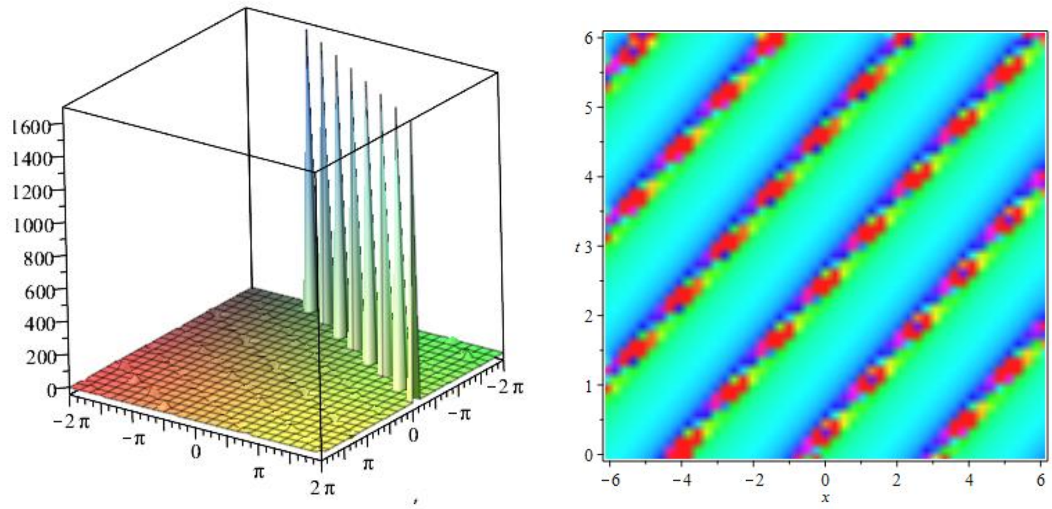

Figure 4. plots of ***Eq. (29)*** with $A = 1, B = 2, \lambda = 1, \mu = 1, y = 0$.

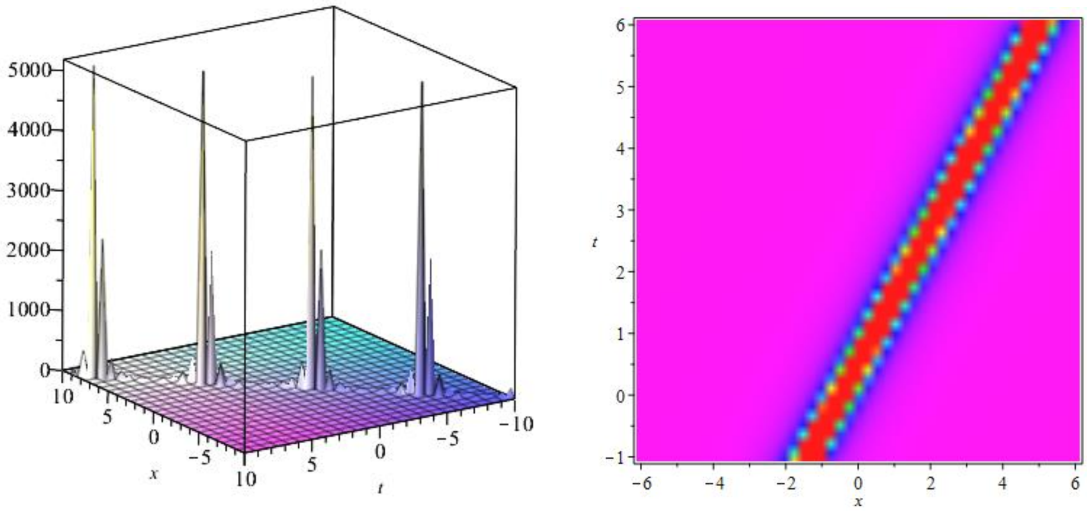

Figure 5. plots of ***Eq. (32)*** with $A = 1, B = 2, \lambda = 2, \mu = 1, y = 0$.

5. Results and Discussion

In this study, some types of solutions for ***Eq. (1)*** were constructed and given a graphical representation. Various kind of graphics of different solutions were drawn in ***Figures 1-5*** which vividly present the soliton wave solution.

By substituting $\delta = \mu = -1$ in ***Eq. (19)***, the solution of ***Eq. (1)*** by kudryashov method will be obtained as follows

$$q(\varepsilon) = \frac{7}{6} - 2\tan h^2 (x + y + \frac{21}{6}\mathrm{t}). \quad \textbf{(33)}$$

The above solution can be obtained by considering $\lambda^2 - 4\mu = 4$ in ***Eq. (27)*** which is the acquired solution by G′/G expansion method. Therefore, the outcoms of the two methods are equivalent to each other.

6. Conclusion

The (2+1)-dimensional Sakovich equation was meticulously investigated in the present work and the exact solution of this equation has been attained fruitfully by the effective Kudryashov and G′/G expansion methods. As it has been disscussed in the article, the solutions obtained from the two methods are equivalent, that is to say, by considering the appropriate values of the parameters, the solution obtained from one method can be generalized to the other one as well. Among the main advantages of the presented methods, the following features can be considered precisely simplicity, directness, reliability and being computerizable. Moreover, Reducing the volume of computer calculations of the used methods compared to similar methods is another merit of it.

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