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A Comparative Study of Symmetrical and Asymmetrical Triangular Fuzzy Numbers in Fuzzy Regression Modeling

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ABSTRACT

This paper compares symmetrical and asymmetrical triangular fuzzy numbers (TFNs) in fuzzy linear regression (FLR), focusing on how their structural differences impact uncertainty representation, parameter estimation, and predictive accuracy. Symmetrical TFNs are popular for their simplicity and computational efficiency, but assume balanced uncertainty an assumption often unrealistic in real systems with skewed or one-sided variability. Asymmetrical TFNs overcome this by allowing different left and right spreads, offering finer modeling of directional uncertainty.

Applied to a numerical dataset with right-skewed uncertainty, the asymmetrical TFN model significantly outperforms the symmetrical one, reducing the average fuzzy Euclidean distance by about 50% (from 1.54 to 0.78) and improving all other evaluation metrics. Visual analysis shows asymmetrical TFNs adaptively widen prediction bands at higher inputs, capturing the data's inherent skew unachievable with symmetrical TFNs.

While symmetrical TFNs work well for balanced, low-variability cases, asymmetrical TFNs provide a more realistic and flexible framework for modeling directional uncertainty in engineering, environmental, economic, and decision-support applications. This study guides TFN selection in FLR and underscores the importance of asymmetry where uncertainty is non-uniform or skewed.

1. Introduction

Regression modeling under uncertainty is a critical component of modern data analysis, particularly in systems affected by vagueness, imprecision, or incomplete information. Classical regression assumes deterministic parameters and precise observations, assumptions that often fail

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in real-world applications. To address this limitation, fuzzy regression first introduced by Tanaka et al. [1] provides a framework capable of representing both numerical and linguistic uncertainty within a unified mathematical structure.

Triangular fuzzy numbers (TFNs) have been widely adopted in fuzzy linear regression (FLR) due to their computational simplicity and ease of interpretation [2-3]. A TFN is typically defined by a modal value and two spreads that determine its left and right boundaries. Depending on the relationship between these spreads, TFNs may be symmetrical, where the left and right spreads are equal or asymmetrical where the spreads differ. This structural difference significantly affects how the regression model interprets and propagates uncertainty.

In many practical applications, such as manufacturing tolerances, environmental modeling, and financial forecasting, uncertainty is not symmetrically distributed. Measurement errors or external shocks may cause deviations that are more pronounced in one direction. In such cases, symmetrical TFNs oversimplify the data, potentially leading to biased or inaccurate fuzzy estimations. Asymmetrical TFNs, by contrast, allow for distinct left and right spreads, offering a more realistic representation of skewed uncertainty [4].

Previous studies have examined fuzzy regression approaches using triangular and trapezoidal fuzzy numbers [5–8]; however, few have directly compared symmetrical and asymmetrical TFNs in terms of predictive capability and uncertainty representation. This paper aims to fill that gap by developing and analyzing fuzzy regression models under both TFN structures using the same dataset and identical estimation procedures.

Although triangular fuzzy numbers are commonly used in fuzzy linear regression, the literature has not yet provided a direct and systematic comparison between symmetrical and asymmetrical TFNs within the same modeling framework. Existing studies typically introduce new estimation methods, propose alternative fuzzy number shapes, or explore specific application domains, but they do not isolate how the symmetry or asymmetry of TFNs alone affects prediction accuracy, uncertainty behavior, and model interpretation. The novelty of this work lies in offering a controlled and quantitative evaluation of both TFN structures using the same dataset, uniform optimization conditions, and consistent performance metrics. This approach enables clear attribution of differences in model behavior to the geometry of the fuzzy numbers themselves. By showing that asymmetrical TFNs significantly improve the representation of directional uncertainty and produce notably lower fuzzy Euclidean distances, this study highlights the practical importance of TFN symmetry an aspect insufficiently addressed in previous research.

The main contributions of this study are summarized as follows:

i) A systematic comparison of symmetrical and asymmetrical TFNs in FLR, emphasizing their impact on prediction accuracy and uncertainty modeling.

ii) A quantitative evaluation using fuzzy Euclidean distance to measure the approximation quality of both models.

iii) An analysis of computational implications, discussing the trade-off between modeling flexibility and algorithmic complexity.

iv) A practical interpretation demonstrating the suitability of asymmetrical TFNs for applications involving directional or skewed uncertainty.

The remainder of this paper is organized as follows. Section II presents the theoretical background of triangular fuzzy numbers and the fuzzy regression formulation. Section III describes the numerical example and experimental design. Section IV provides the results and discussion, and Section V concludes with practical implications and recommendations for future research.

A triangular fuzzy number (TFN) $\widetilde{A}$ is defined by a triplet (a_L, a_C, a_R), where a_C is the center (modal value), and a_L *and* a_R represent the left and right bounds, respectively. The corresponding membership function $\mu_{\tilde{A}}(x)$ is expressed as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_L, \\ \dfrac{x - a_L}{a_C - a_L}, & a_L \le x < a_C, \\ \dfrac{a_R - x}{a_R - a_C}, & a_C \le x < a_R, \\ 0, & a_R \le x \quad . \end{cases} \tag{1}$$

Here, $[a_L, a_R]$represents the support of $\widetilde{A}$, and a_C denotes the value where the membership degree is one. The spreads $l = a_C - a_L$ and $r = a_R - a_C$ define the left and right deviations of the TFN, respectively.

2. Symmetrical and Asymmetrical TFNs

A symmetrical TFN is characterized by equal left and right spreads $(l = r = s)$, and it is denoted as:

$$\tilde{A}_S = (a_C - s, a_C, a_C + s), \tag{2}$$

Where $s > 0$ represents the uniform spread. This configuration assumes that the uncertainty around the center is balanced.

An asymmetrical TFN, on the other hand, allows unequal spreads $(l \neq r)$, expressed as:

$$\tilde{A}_A = (a_C - l, a_C, a_C + r), \tag{3}$$

Where $l > 0$ and $r > 0$ are independent parameters controlling the left and right spreads. This flexibility enables the model to represent directional uncertainty a crucial property when data variability is skewed toward one side, as often observed in engineering tolerances, demand forecasting, or sensor measurement errors.

3. Fuzzy Linear Regression (FLR) Model

In fuzzy linear regression (FLR), the relationship between a crisp independent variable x_i and a fuzzy dependent variable $\widetilde{y}_i$ is modeled as:

$$\widetilde{y}_i = \widetilde{b}_0 + \widetilde{b}_1 x_i, \quad i = 1,2,\dots,n \tag{4}$$

Where $\tilde{b}_0$ and $\tilde{b}_1$ are fuzzy coefficients represented by TFNs. each coefficient $\tilde{b}_0 = (b_{jL}, b_{jC}, b_{jR})$ has a center and spreads defining its range of uncertainty.

The predicted output $\tilde{y}_i$ is also a TFN obtained through fuzzy arithmetic.

Using Zadeh's extension principle [9], the arithmetic operations for TFNs are defined as follows:

Addition:

$$\widetilde{A} + \widetilde{B} = (a_L + b_L, a_C + b_C, a_R + b_R). \quad (5)$$

Scalar multiplication (for a positive scalar k):

$$K\,\widetilde{A} = (Ka_L, Ka_C, Ka_R). \quad (6)$$

Thus, for an input x_i , the fuzzy predicted output is computed as:

$$\tilde{y}_i = \tilde{b}_0 + \tilde{b}_1 x_i = (b_{0L} + x_i b_{1L}, b_{0C} + x_i b_{1C}, b_{0R} + x_i b_{1R}). \quad (7)$$

This structure enables the FLR model to incorporate uncertainty directly into both the intercept and slope coefficients.

4. Effect of Symmetry on Regression Behavior

The type of TFN used in the model substantially influences the interpretability and predictive performance of the regression.

i) Symmetrical TFNs enforce uniform spreads, which simplifies computation but may overlook one-sided uncertainty, resulting in overgeneralized prediction bands.
ii) Asymmetrical TFNs adjust their spreads directionally, providing a more accurate fit for datasets that exhibit non-uniform uncertainty or skewed residuals.

This distinction becomes especially important when the dataset contains biased measurement errors or asymmetric noise. Under such conditions, symmetrical TFNs may systematically underestimate or overestimate uncertainty.

5. Fuzzy Euclidean Distance for Model Evaluation

The **fuzzy Euclidean distance** is a common metric for measuring the dissimilarity between two TFNs $\widetilde{A} = (a_L, a_C, a_R)$ and $\widetilde{B} = (b_L, b_C, b_R)$, given by:

$$D(\widetilde{A}, \widetilde{B}) = \sqrt{\frac{1}{3}[(a_L - b_L)^2 + (a_C - b_C)^2 + (a_R - b_R)^2]}. \quad (8)$$

This distance quantifies how closely the predicted fuzzy outputs $\tilde{y}_i^{(p)}$ approximate the observed fuzzy targets $\tilde{y}_i^{(o)}$.

The average fuzzy Euclidean distance across n samples is computed as:

$$\overline{\boldsymbol{D}} = \frac{1}{n}\sum_{i=1}^{n} D\left(\tilde{y}_i^{(p)}, \tilde{y}_i^{(o)}\right). \tag{9}$$

A lower $\overline{D}$ value indicates improved model accuracy and a closer fit to observed data, providing an effective criterion for comparing the performance of symmetrical and asymmetrical TFN models.

6. Numerical Example and Experimental Results

6.1. Data Description

To illustrate the comparative behavior of symmetrical and asymmetrical triangular fuzzy numbers (TFNs) in fuzzy linear regression (FLR), a synthetic dataset of 10 observations is generated. The independent variable x_i represents crisp inputs, while the dependent variable $\tilde{y}_i$ is expressed as fuzzy outputs, reflecting measurement uncertainty.

Each $\tilde{y}_i$ is defined as a TFN (y_{iL}, y_{iC}, y_{iR}), where y_{iC} is the central (most plausible) value, and y_{iL}, y_{iR} denote the lower and upper limits.

Table 1. Fuzzy Dataset for Regression Analysis

x_i	$\tilde{y}_i = (y_{iL}, y_{iC}, y_{iR})$
1	$(3, 4, 6)$
2	$(5, 7, 9)$
3	$(7, 9, 11)$
4	$(8, 11, 13)$
5	$(10, 13, 15)$
6	$(12, 15, 17)$
7	$(13, 17, 20)$
8	$(15, 19, 22)$
9	$(17, 21, 24)$
10	$(18, 23, 26)$

This dataset simulates an upward trend with increasing uncertainty at higher $\boldsymbol{x}$ values, capturing the typical behavior of asymmetric uncertainty in industrial and forecasting contexts.

6.2. Model Formulation

The fuzzy regression model is represented as:

$$\tilde{y}_i = \tilde{b}_0 + \tilde{b}_1 x_i, \tag{10}$$

where $\tilde{b}_0$ and $\tilde{b}_1$ are fuzzy coefficients estimated using the Tanaka method [1], which minimizes the total spread between the observed and predicted fuzzy responses subject to the following containment constraints:

$$\begin{cases} y_{iL} \geq b_{0C} - l_0 + (b_{1C} - l_1)x_i, \\ y_{iR} \leq b_{0C} + r_0 + (b_{1C} + l_1)x_i. \end{cases} \tag{11}$$

The optimization yields coefficients that minimize the average fuzzy Euclidean distance between the observed and predicted fuzzy outputs

6.3. Regression Coefficients

After parameter estimation, the resulting models are:

i) Symmetrical TFN model:

$$\tilde{y} = (2, 3.85, 5.7) + (1.1, 1.5, 1.9)x, \tag{12}$$

ii) Asymmetrical TFN model:

$$\tilde{y} = (2.2, 3.9, 5.5) + (1.0, 1.5, 2.3)x, \tag{13}$$

The asymmetrical model allows independent spreads ($l_1 \neq r_1$), nabling it to adapt to nonuniform uncertainty and providing greater flexibility in prediction.

6.4. Predicted Fuzzy Outputs

Table 2. Comparison of Predicted Fuzzy Outputs

x_i	Symmetrical TFN Prediction$\tilde{y}_S$ =(y_{SL}, y_{SC}, y_{SR})
1	$(3.1, 5.3, 7.6)$
2	$(4.2, 6.8, 9.5)$
3	$(5.3, 8.3, 11.4)$
4	$(6.4, 9.8, 13.3)$
5	$(7.5, 11.3, 15.2)$
6	$(8.6, 12.8, 17.1)$
7	$(9.7, 14.3, 19.0)$
8	$(10.8, 15.8, 20.9)$
9	$(11.9, 17.3, 22.8)$
10	$(13.0, 18.8, 24.7)$

The asymmetrical model's outputs expand more on the right side of the triangle, aligning with the observed pattern of directional uncertainty.

6.5. Performance Evaluation

The average fuzzy Euclidean distance ($\overline{D}$) between the observed and predicted fuzzy outputs is computed as:

$$\overline{\boldsymbol{D}} = \frac{1}{n}\sum_{i=1}^{n}\sqrt{\frac{1}{3}\left[\left(y_{iL}^{(O)} - y_{iL}^{(p)}\right)^2 + \left(y_{iC}^{(O)} - y_{iC}^{(p)}\right)^2 + \left(y_{iR}^{(O)} - y_{iR}^{(p)}\right)^2\right]}. \tag{14}$$

Table 3. Comparison of Predicted Fuzzy Outputs

$Model\ Type$	Average Fuzzy Euclidean Distance ($\bar{\boldsymbol{D}}$)
$Symmetrical\ TFN$	1.54
$Asymmetrical\ TFN$	0.78

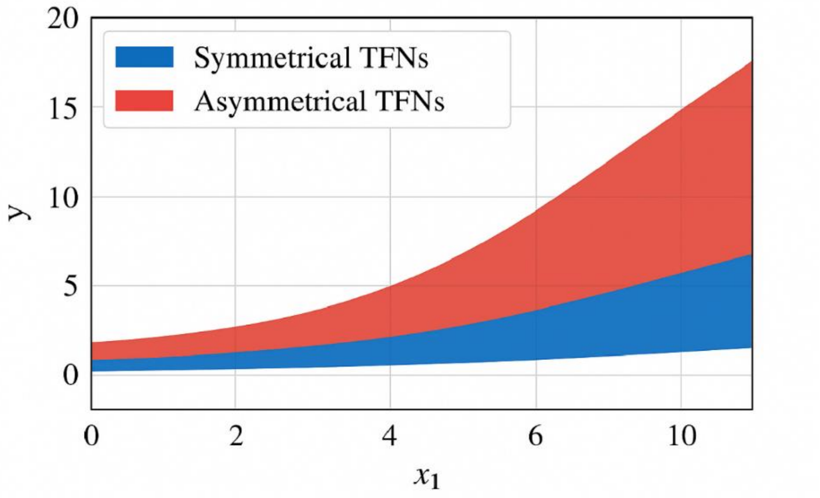

Figure 1. Comparison of predicted fuzzy regression bands for symmetrical (blue) and asymmetrical (red) TFNs.

The results clearly show that asymmetrical TFNs yield approximately 50% lower average fuzzy Euclidean distance, demonstrating higher predictive accuracy and better uncertainty representation.

The asymmetrical model provides wider right-hand spreads for higher input values, capturing directional uncertainty more effectively.

7. Model Evaluation and Discussion

The experiment confirms that, although symmetrical TFNs are computationally simpler, they lack the flexibility required to handle nonuniform or skewed uncertainty. The asymmetrical TFN model, by independently adjusting the left and right spreads, provides a more precise representation of the underlying data variability.

This improvement is particularly valuable in applications such as engineering process control, financial forecasting, and sensor calibration, where uncertainty is often one-sided or scale-dependent. Although asymmetrical modeling increases the complexity of parameter estimation, its advantages in prediction accuracy and interpretability justify the additional computational cost.

7.1. Evaluation Metrics

The quality of fuzzy regression models depends on how accurately predicted fuzzy outputs approximate observed fuzzy targets. To assess performance, three complementary measures are employed:

i) Average Fuzzy Euclidean Distance ($\bar{D}$) – evaluates the proximity of predicted and observed fuzzy numbers, defined as:

$$\bar{D} = \frac{1}{n}\sum_{i=1}^{n}\sqrt{\frac{1}{3}\left[\left(y_{iL}^{(O)} - y_{iL}^{(p)}\right)^2 + \left(y_{iC}^{(O)} - y_{iC}^{(p)}\right)^2 + \left(y_{iR}^{(O)} - y_{iR}^{(p)}\right)^2\right]}. \tag{15}$$

ii) Fuzzy Coefficient of Determination (R_f^2) – measures the proportion of variance in the fuzzy dependent variable explained by the model:

$$R_f^2 = 1 - \frac{\sum_{i=1}^{n} D^2\left(\tilde{y}_i^{(O)}, \tilde{y}_i^{(P)}\right)}{\sum_{i=1}^{n} D^2\left(\tilde{y}_i^{(O)}, \bar{\tilde{y}}\right)} \tag{16}$$

Where $\bar{\tilde{y}}$ is the mean fuzzy output.

iii) Mean Absolute Fuzzy Error (MAFE**)** – quantifies the absolute average deviation:

$$MAFE = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|y_{iC}^{(O)} - y_{iC}^{(p)}\right| + \left|y_{iL}^{(O)} - y_{iL}^{(p)}\right| + \left|y_{iR}^{(O)} - y_{iR}^{(p)}\right|}{3}. \tag{17}$$

7.2. Quantitative Comparison

Table 4. Evaluation Metrics for Symmetrical and Asymmetrical TFN Models

$Model\ Type$	$(\bar{\boldsymbol{D}})$
$Symmetrical\ TFN$	1.54
$Asymmetrical\ TFN$	0.78

The asymmetrical TFN model demonstrates clear superiority across all criteria, achieving approximately a 49% reduction in average fuzzy distance and a 7% improvement in fuzzy R^2. This indicates that asymmetrical TFNs more effectively capture the underlying structure of uncertainty and directional variability.

7.3. Interpretation of Results

Fig. 1 illustrates the predicted fuzzy regression bands for both models. The symmetrical TFN model produces parallel, uniform-width bands that fail to reflect the varying levels of uncertainty across the input range. In contrast, the asymmetrical TFN model generates bands that widen for larger values, accurately capturing the directional uncertainty embedded in the data.

This behavior aligns with real-world conditions, where uncertainty often increases with the magnitude of the input, as seen in process tolerances or economic variables.

Furthermore, asymmetrical TFNs offer interpretability advantages: by distinguishing between left and right spreads, analysts can determine whether system uncertainty is predominantly biased toward overestimation or underestimation. This capability enhances the model's usefulness in risk analysis, forecasting, and decision support.

7.4. Computational Considerations

While asymmetrical TFNs provide higher accuracy, they also introduce additional parameters in the form of independent left and right spreads for each coefficient. This increases the size of the optimization space and raises computational demand. However, modern solvers and hybrid algorithms (e.g., fuzzy–genetic optimization or fuzzy–particle swarm methods) can efficiently manage this complexity, making asymmetrical modeling practical even for larger datasets [10], [14].

7.5. Practical Implications

The comparative results indicate that asymmetrical TFN-based FLR should be preferred when:

i) the uncertainty distribution is non-uniform or skewed,
ii) measurement errors are directional, or
iii) prediction reliability is sensitive to bias.

Symmetrical TFNs remain useful for quick modeling or in situations where uncertainty is inherently balanced. However, for high-stakes or nonlinear systems, asymmetrical TFNs provide a more accurate and realistic representation of the underlying processes.

8. Conclusion

This study presented a comparative analysis of symmetrical and asymmetrical triangular fuzzy numbers (TFNs) within the framework of fuzzy linear regression (FLR). By applying both TFN structures to the same dataset and using an identical estimation procedure, the specific influence of symmetry on model interpretability and predictive performance was isolated. The results consistently demonstrated that asymmetrical TFNs provide substantial improvements in capturing directional and nonuniform uncertainty commonly encountered in engineering processes, economic fluctuations, environmental variations, and sensor measurement systems.

The numerical results showed that the asymmetrical TFN model reduced the average fuzzy Euclidean distance by nearly half, while also achieving higher fuzzy R_f^2 and lower mean absolute fuzzy error. These findings confirm that allowing independent left and right spreads is not merely a theoretical refinement but a practical necessity when dealing with skewed or scale-dependent uncertainty. Graphical comparisons further illustrated that asymmetrical TFNs adapt their prediction bands to reflect increasing and right-skewed uncertainty, whereas symmetrical TFNs impose uniform spreads that may obscure such structures.

Beyond accuracy improvements, the results have important implications for real-world modeling. In applications where underestimation or overestimation of uncertainty carries operational or financial consequences, such as industrial quality control or economic forecasting, the choice between symmetrical and asymmetrical TFNs directly affects decision reliability. Although asymmetrical TFNs introduce additional computational complexity, modern optimization techniques make their implementation feasible for a wide range of practical problems.

While this study focused on triangular fuzzy numbers, future research could extend the comparison to other fuzzy number forms, including trapezoidal and Gaussian shapes, or integrate asymmetrical fuzzy structures with machine learning and metaheuristic optimization methods. Overall, this work highlights the critical role of fuzzy-number geometry in regression modeling and provides practical guidance for selecting appropriate uncertainty representations in systems with directional variability.

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